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all values of h corresponding to $\rho(t, t+h)<r$, and hence $F(h)$, being an always increasing function in the interval, can vanish at most but once. We omit the simple proof that if $\delta<r$ and if r is small enough $F(h)$ *does* vanish once.

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DIVISIONS OF AN ANGLE INTO EQUAL PARTS BY MEANS OF A TRANSCENDENTAL CURVE.

By J. S. BROWN, San Marcos, Texas.

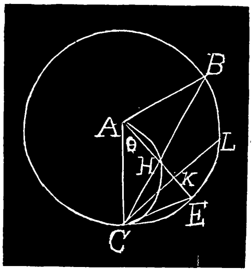
The problem of trisecting an angle by means of the cardioid is capable of a much more general setting as follows:

Given CAE an isosceles triangle, $AC=AE$, angle $A=\theta$; also given a point H in AE such that angle $HCE=n.\theta$. Required, the locus of H if AC is fixed and AE rotates in the plane about A. E will describe a circle with A as center and AE as radius, as indicated in the figure.

Draw CL perpendicular to AE; angle $LCE=\frac{1}{2}\theta$; $CK=CH\cos(n-\frac{1}{2})\theta$, and $CK=AC\sin\theta$. Then

$$CH=\frac{AC\sin\theta}{\cos(n-\frac{1}{2})\theta}, \tag{1}$$

which is the polar equation of the locus of H when C is taken as origin and AC is the axis of reference.



By assigning the successive values $\frac{1}{2}$, 1, $1\frac{1}{2}$, etc., to n the particular equations of the curves by means of which subdivisions one-half, one-third, etc., of an angle can be constructed, are found.

As an example, suppose it is desired to find the equation of the curve by means of which an angle with A as vertex and AC as one side, can be trisected; for example, angle BAC.

It is evident that in this case $n=1$. Substituting this value of n in the general equation, we find

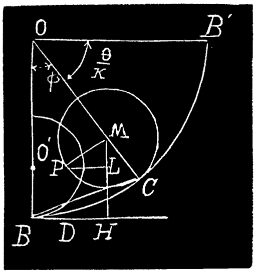
$$CH=\frac{AC\sin\theta}{\cos\frac{1}{2}\theta}=\frac{AC.2\sin\frac{1}{2}\theta.\cos\frac{1}{2}\theta}{\cos\frac{1}{2}\theta}=AC.2\sin\frac{1}{2}\theta=2AC\sin\frac{1}{2}\theta. \tag{2}$$

Suppose AHC to be the curve for this value of n, and let H be the point where BC cuts this curve. Draw AHE. The angle $CAE=\frac{1}{3}CAB$, for angle $BCE=\theta$ and angle $BAE=2\theta$.

Evidently, this curve corresponding to (2) is the cardioid, though the form of the solution here used is quite different from the usual method of trisection by the cardioid.

In this manner the curve corresponding to any value of n as above indicated can be found. By means of an instrument constructed on the principle which governs the relative rates of motion of the hands of a clock, any of these curves can be accurately drawn.

The hypocycloid curve may also be used for the purpose of general equi-section of an angle, as appears in the following:



Given OB the radius of a circle, and suppose $O'B=r=\frac{OB}{k.n}$ where $\frac{\theta}{360}=\frac{1}{n}$, and k is the number of equal parts into which it is required to divide the angle θ.

B and B' are two consecutive points common to the hypocycloid and the circumference of the circle whose center is O, the rolling circle having the radius $O'B=r$. It is evident that the angle $BOB'=\theta/k$. To find the equation of the curve, let B be the origin, BH the X-axis, BO the Y-axis, ϕ the angle BOC, P a point on the curve, $x=BD$ and $y=PD$. Then $y=MF+FH-ML=r+FH-ML$.

But $ML=r\cos(kn-1)\phi$, $FH=(BD+DH)\tan\frac{1}{2}\phi=(x+DH)\tan\frac{1}{2}\phi$, $DH=PL=r\sin(kn-1)\phi$.

Hence, $y=r+[x+r\sin(kn-1)\phi]\tan\frac{1}{2}\phi-r\cos(kn-1)\phi$, the equation of the hypocycloid, in which it may be noted that angle $PMC=k.n.\phi$.

While the equation is general, the construction of the curve can be made only when the ratio of the given angle to the perigon is known.

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DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

ALGEBRA.

290. Proposed by G. I. HOPKINS, M. A., Manchester, New Hampshire.

A and B are 45 miles apart and travel towards each other. A goes one mile the first day, three miles the second day, five miles the third day, and so on. B goes two miles the first day, four the second, six the third, and so on. In how many days will they meet? What interpretation is to be placed upon the negative value of n?

Solution by THEODORE L. DeLAND, Treasury Department, Washington, D. C.

Let n=the required number of days for A and B to meet. When a,